where $\hat{r} = r/D$ is the normalized distance downrange.

The cross correlation between the radial gravity disturbances at launch and at $\hat{r}$ is given by

$$R_{gg}(0,\hat{r}) = \frac{\sigma_0^2 (1+3\hat{r}/2+3\hat{r}^2/8-\hat{r}^3/16)(1+\hat{r})^2}{(1+\hat{r}+\hat{r}^2/2)^{7/2}} \tag{17}$$

Using the first-order model described above, a satisfactory fit is found for $P_0 = \sigma_0^2$, $P_\infty = 0.0751\ \sigma_0^2$, and $\beta = 2.1048$.†

The fit provided by these parameters is shown in Figs. 1 and 2. Figure 1 illustrates the comparison between Eq. (16) for the AWN model and Eq. (8) for the state-space model. Figure 2 compares Eqs. (17) and (9).

The model fit shown was chosen as a compromise between appropriate decay of the disturbance variance with altitude and decorrelation with shift distance. The former is controlled at small shifts by β and at large shifts by P_∞, as is evident from Eq. (8). Figure 1 shows that at large shifts P_∞ introduces a "tail" into Eq. (8) which deviates from Eq. (16). However, reducing P_∞ causes the cross correlation R to approach unity for all shifts, so the "tail" in Eq. (8) trades off against an acceptable decay of cross correlation shown in Fig. 2. In both Figs. 1 and 2 it can be seen that a good match has been achieved in the significant features of the desired statistical behavior.

Conclusions

Explicit equations for the nonstationary transient behavior of a stochastic linear dynamic system have been derived in a closed form. These equations have been applied to model the statistics of the gravity disturbance along a missile trajectory. The examples presented illustrate that a reasonable fit can be achieved to the statistics predicted by more theoretical gravity models. The nonstationary state-space model, however, has the advantage that it can be implemented directly in Kalman filtering or smoothing software used for processing of missile system tests.

Appendix

This appendix shows that Eq. (4) correctly describes the statistical transient of a linear dynamic system. It provides a closed form free of integrals. The solution to Eq. (1) can be calculated using standard methods to be

$$x(t) = \Phi(t)x(0) + \int_0^t \Phi(t-\sigma)w(\sigma)\mathrm{d}\sigma \tag{A1}$$

Computing the expectation in Eq. (4) and using Eqs. (2) and (3) to simplify the resulting integral yields

$$\begin{aligned} C(t_1,t_2) &= \Phi(t_1)P_0\Phi(t_2) \\ &\quad + \Phi(t_1)\int_0^d \Phi(-\sigma)Q\Phi^T(-\sigma)\mathrm{d}\sigma\Phi^T(t_2) \end{aligned} \tag{A2}$$

Comparing Eqs. (A2) and (4) it is seen that these expressions are equivalent if

$$\int_0^\tau \Phi(-\sigma)Q\Phi^T(-\sigma)\mathrm{d}\sigma \overset{?}{=} \Phi(-\tau)P_\infty\Phi^T(-\tau) - P_\infty \tag{A3}$$

Equation (A3) is clearly satisfied for $\tau = 0$, in which case both sides are zero.

Next, the derivatives of both sides of Eq. (A3) with respect to τ are shown to be equal, as follows,

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\tau}\int_0^\tau \Phi(-\sigma)Q\Phi^T(-\sigma)\mathrm{d}\sigma &= \Phi(-\tau)Q\Phi^T(-\tau) \\ &= \Phi(-\tau)[-FP_\infty - P_\infty F^T]\Phi^T(-\tau) \\ &= -\Phi(-\tau)FP_\infty\Phi^T(-\tau) - \Phi(-\tau)P_\infty F^T\Phi^T(-\tau) \\ &= \frac{\mathrm{d}}{\mathrm{d}\tau}[\Phi(-\tau)P_\infty\Phi^T(-\tau)] \end{aligned} \tag{A4}$$

Thus, both sides of Eq. (A3) satisfy the same homogeneous differential equation and match at the boundary $\tau = 0$. The uniqueness of the solution to the differential equation establishes the equality in Eq. (A3), and thereby shows that Eqs. (A2) and (4) are equivalent expressions for $C(t_1,t_2)$.

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Active Attitude Control of a Spinning Symmetrical Satellite in an Elliptic Orbit

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Introduction

THE equations describing the attitude motion of a spinning symmetrical satellite in an elliptic orbit and subject to gravity torques are nonautonomous, nonlinear, ordinary differential equations. The nonautonomous nature of the equations is due to the dependence of the spin rate and the gravity torque on the time periodic orbital motion.

Various authors have considered the stability of these equations. Specifically, Kane and Barba[1] linearized the attitude equations to obtain a set of linear ordinary differential equations with periodic coefficients. These authors then determined the stability of these equations using Floquet theory. Wallace and Meirovitch[2] used a perturbation technique to determine stability. In both references, stability was determined as a function of the orbital eccentricity and the satellite's spin rate and inertia ratio. The results of these investigations showed that for fixed spin rate, and inertia ratios, stability was adversely affected by orbital eccentricity.

This Note extends the work of Refs. 1 and 2 to include the active control of the satellite. The control task is complicated by the time-dependent nature of the coefficients of the linearized equations of motion. However, since the coefficients are

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periodic functions of time, it can be shown that the systems equations can always be transformed into constant coefficient equations. This transformation uses the time periodic Floquet modes (eigenvectors) to define a new set of modal coordinates, in terms of which the system has constant coefficients. For a controllable system, it is shown that either constant or time varying gains can always be found which stabilize an unstable case or enhance the stability of a stable case.

The conditions for the modal control of a set of linear first-order ordinary differential equations with periodic coefficients and a single control input are derived in this Note. A technique for selecting the gains necessary to produce a change in the stability of a single mode, by a prescribed amount, is developed. The procedure involved is very similar to modal control of constant coefficient systems. However, the modal matrix is time periodic in the present analysis and must be obtained by means of numerical analysis.

The control of an unstable satellite is accomplished as an example of the current method. The control is shown to affect only those modes of the satellite which are unstable.

Problem Formulation

The linearized equations for the attitude motion of a spinning symmetrical satellite in an elliptic orbit of eccentricity ϵ and semimajor axis a are developed in Ref. 1.

These equations, written in first-order form, are

$$\bar{x}' = A(\tau)\bar{x} \tag{1}$$

where the prime denotes derivatives with respect to the dimensionless time τ ($\tau = t$/mean motion) and

$$\bar{x}^T = [\theta_1, \theta_2, \theta_1', \theta_2'] \tag{2}$$

The angles θ_1 and θ_2 locate the spacecraft's spin axis with respect to an orbital reference frame. In addition, the nonzero elements of the matrix $A(\tau)$ are given by

$$a_{31} = (1-\epsilon^2)/\zeta^4 - (1+\alpha)(1+K)(1-\epsilon^2)^{1/2}/\zeta^2$$

$$a_{42} = a_{31} - 3K/\zeta^3$$

$$a_{32} = -a_{41} = -2(1-\epsilon^2)^{1/2}\zeta'/\zeta^3$$

$$a_{34} = -a_{43} = 2(1-\epsilon^2)^{1/2}/\zeta^2 - (1+\alpha)(1+K)$$

$$a_{13} = a_{24} = 1 \tag{3}$$

where $\zeta = r/a$ is the dimensionless orbital radius; $K = J/I - 1$ is one less than the ratio of the satellite's spin axis to the transverse axis inertias; and α is a dimensionless spin parameter defined by

$$\alpha = (\dot{\theta}_3 + \dot{\theta})/n - 1$$

where n is the mean motion, and $\dot{\theta}_3 + \dot{\theta}$ represents the constant equilibrium spin rate. In order to control the satellite, a single control is assumed such that the state equations including the control are given by

$$\bar{x}' = A(\tau)\bar{x} + \bar{b}u \tag{4}$$

where, for convenience, $\bar{b}^T = (0,0,1,1)$. This completes the formulation of the equations of motion. These equations have periodic coefficients as described by Eqs. (3). The period of $A(\tau)$ is the orbital period T. The stability of the uncontrolled motion of the system described by Eq. (4) was determined in Refs. 1 and 2. This Note will consider the control of the system of Eq. (4).

Active Control

Floquet Theory

For a linear system any solution may be written as a linear combination of n linearly independent solutions, and system stability is determined by the stability of the n solutions. When the coefficients are periodic in time, as in the present case, system stability may be determined by studying the behavior of n linearly independent solutions over a time interval equal to the period of the coefficients. This fact is a main result of Floquet theory. To briefly review Floquet theory consider the uncontrolled system

$$\bar{x}' = A(\tau)\bar{x} \tag{5}$$

where $A(\tau) = A(\tau + T)$. Floquet theory[3,4] states that every fundamental matrix for Eq. (5) can be written in the form

$$\phi(\tau) = P(\tau)e^{\Gamma\tau} \tag{6}$$

where $P(\tau) = P(\tau + T)$ and Γ is a constant matrix. Furthermore, if the similarity transformation which reduces Γ to Jordan form is B, then

$$\psi(\tau) = B^{-1}\phi(\tau)B = F(\tau)e^{J\tau} \tag{7}$$

is also a fundamental matrix for Eq. (5) and $F(\tau) = F(\tau + T)$. The eigenvalues of Γ, ρ_i are the diagonal elements of J, and are termed the characteristic exponents. From Eq. (7) it is apparent that if all of the characteristic exponents have negative real parts, the system is stable. Conversely, if any of the exponents have positive real parts, it is unstable. The characteristic exponents may be calculated if the matrix Γ can be determined. Evaluating Eq. (6) at the end of one period and multiplying by $\phi(0)^{-1}$ yields

$$C = \phi^{-1}(0)\phi(T) = e^{\Gamma T} \tag{8}$$

The matrix C is called the monodromy matrix and its eigenvalues λ_i are related to those of Γ by

$$\rho_i = (1/T)\ln\lambda_i \qquad i = 1,\ldots,n \tag{9}$$

The λ_i are the eigenvalues of C and are called the characteristic multipliers. The stability of the system given by Eq. (5) can then be determined from the λ_i. Specifically, if the magnitudes of all λ_i are less than 1 the system is asymptotically stable. However, if any λ_i has a magnitude greater than 1 the system is unstable. If the magnitude of λ_i is equal to 1 the system is stable if the multiplicity of λ_i is equal to its nullity and unstable otherwise. In summary, Floquet theory states that the stability of a set of linear ordinary differential equations with periodic coefficients may be determined by forming the monodromy matrix and examining its eigenvalues.

Modal Control

In the preceding section the use of Floquet theory in analyzing stability was reviewed. The extension of this theory to the active control problem will now be presented. Consider the transformation from the physical coordinates $\bar{x}(\tau)$ to modal variables $\bar{\eta}(\tau)$ given by

$$\bar{x}(\tau) = F(\tau)\bar{\eta}(\tau) \tag{10}$$

where $F(\tau)$ is defined by Eq. (7). Writing Eq. (4) in terms of $\bar{\eta}(\tau)$ yields

$$\bar{\eta}'(\tau) = F^{-1}(\tau)[A(\tau)F(\tau) - F'(\tau)]\bar{\eta}(\tau) + F^{-1}(\tau)\bar{b}u \tag{11}$$

Recalling that

$$\psi'(\tau) = A(\tau)\psi(\tau) \tag{12a}$$

where

$$\psi(\tau)=F(\tau)e^{J\tau} \tag{12b}$$

Equation (11) reduced to

$$\bar{\eta}'(\tau)=J(\tau)\bar{\eta}(\tau)+\bar{r}(\tau)u \tag{12c}$$

The vector $\bar{r}(\tau)$ is the mode controllability vector and each mode η_i is controllable if the corresponding r_i is non-zero. The periodic matrix $F(\tau)$ satisfies the matrix differential equation

$$F'(\tau)=A(\tau)F(\tau)-F(\tau)J \tag{13}$$

Matrices $A(\tau)$ and J are said to be kinematically similar. A periodic matrix $A(\tau)$ is always kinematically similar to a constant matrix.[4] It is interesting to note that for constant matrices Eq. (13) reduces to the familiar form. Similarly, by noting that the product of F and its inverse is the identity matrix, F^{-1} is given by the solution of

$$[F^{-1}(\tau)]'=-F^{-1}(\tau)A(\tau)+JF^{-1}(\tau) \tag{14}$$

Since both $F(\tau)$ and $F^{-1}(\tau)$ are periodic matrices, they can be determined by integrating the matrix equations (13) and (14) over one period. The initial condition matrix $F(0)$ is the matrix whose columns are the eigenvectors of the monodromy matrix C.

Single-Mode Control

Equations (12a-c) are uncoupled constant coefficient equations with periodic control terms. Considering the case where all of the characteristic exponents are unique, the η_i satisfy equations of the form

$$\eta_i'(\tau)=\rho_i\eta_i(\tau)+r_i(\tau)u \qquad i=1,\ldots,n \tag{15}$$

The stability of each of the uncontrolled modes is, of course, determined by the values ρ_i. Assume that ρ_i is positive and, hence, the mode associated with ρ_i is unstable. In order to stabilize this mode a scalar control of the form

$$u(\tau)=k_i(\tau)\eta_i(\tau) \tag{16}$$

is considered. The closed-loop system equations with this control added become

$$\bar{\eta}'_{\mathrm{CL}}(\tau)=\begin{bmatrix} \rho_1 & 0 & \ldots & r_1(\tau)k_i(\tau) & \ldots & 0 \\ 0 & \rho_2 & \ldots & r_2(\tau)k_i(\tau) & \ldots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \ldots & \rho_i+r_i(\tau)k_i(\tau) & \ldots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \ldots & r_n(\tau)k_i(\tau) & \ldots & \rho_n(\tau) \end{bmatrix}\bar{\eta}_{\mathrm{CL}} \tag{17}$$

Except for ρ_i the characteristic exponents for the closed-loop system are those of the uncontrolled system. The new characteristic exponent $\rho_{i_{\mathrm{CL}}}$ can be determined by considering the differential equation for $\eta_{i_{\mathrm{CL}}}$, namely,

$$\eta'_{i_{\mathrm{CL}}}(\tau)-[\rho_i+r_i(\tau)k_i(\tau)]\eta_{i_{\mathrm{CL}}}(\tau)=0 \tag{18}$$

The solution to this equation is given by

$$\eta_{i_{\mathrm{CL}}}(\tau)=\eta_{i_{\mathrm{CL}}}(0)\exp\left\{\int_0^\tau [\rho_i+r_i(\xi)k_i(\xi)]\mathrm{d}\xi\right\} \tag{19}$$

From the definition of characteristic exponents $\rho_{i_{\mathrm{CL}}}$ is therefore given by

$$\rho_{i_{\mathrm{CL}}}=\rho_i+(1/T)\int_0^T r_i(\xi)k_i(\xi)\mathrm{d}\xi \tag{20}$$

It remains to be shown that the gain $k_i(\xi)$ can be picked such that $\rho_{i_{\mathrm{CL}}}$ can be chosen to be any desired value. In order to show that such is the case, recall that the elements of the mode controllability vector $\bar{r}(\tau)$ are periodic functions of time and can therefore be expressed in terms of infinite Fourier series. That is,

$$r_i(\xi)=a_{0_i}+\sum_{n=1}^{\infty}a_{ni}\cos\frac{2n\pi}{T}\xi+\sum_{n=1}^{\infty}b_{ni}\sin\frac{2n\pi}{T}\xi \tag{21}$$

Consider first the case where the coefficient a_{0_i} is non-zero. The gain k_i in this case is chosen to be a constant and $\rho_{i\mathrm{CL}}$ is given by

$$\rho_{i_{\mathrm{CL}}}=\rho_i+a_{0_i}k_i \tag{22}$$

The gain k_i may now be chosen to yield the desired $\rho_{i_{\mathrm{CL}}}$. If the coefficient a_{0_i} is zero, the gain k_i can be chosen in the form

$$k_i(\xi)=C_i\cos(2n\pi/T)\xi \tag{23a}$$

or

$$k_i(\xi)=C_i\sin(2n\pi/T)\xi \tag{23b}$$

where n is the index of a non-zero Fourier coefficient of $r_i(\xi)$. Using these gains in Eq. (20) yields

$$\rho_{i_{\mathrm{CL}}}=\rho_i+(C_ia_{ni}/2) \tag{24a}$$

or

$$\rho_{i_{\mathrm{CL}}}=\rho_i+(C_ib_{ni}/2) \tag{24b}$$

Therefore, given a_{ni} or b_{ni}, the constant C_i can be chosen such that $\rho_{i_{\mathrm{CL}}}$ has the desired value.

In order to implement this control in terms of the state vector $\bar{x}$, it is noted that

$$u=\bar{k}^T\bar{\eta}=\bar{k}^TF^{-1}(\tau)\bar{x} \tag{25}$$

where $\bar{k}^T=[0\ldots k_i\ldots 0]$ is the gain matrix. For the purposes of the current study it is assumed that the state $\bar{x}$ is available and, hence, Eq. (25) can be implemented. The closed-loop state equations therefore become

$$\bar{x}'_{\mathrm{CL}}=[A(\tau)+\bar{k}^T(\tau)F^{-1}(\tau)]\bar{x}_{\mathrm{CL}} \tag{26}$$

The characteristic exponents for the closed-loop equations are identical to those of the open-loop equations with the exception of ρ_i, which is changed to $\rho_{i_{\mathrm{CL}}}$.

In summary, a scalar control can always be found for a controllable mode such that just that mode's characteristic exponent is changed. The gain necessary to provide this change may either be constant or periodic depending upon the form of the Fourier series representing r_i. The method requires numerically integrating n scalar equations n times to produce F^{-1}, forming r_i in terms of a Fourier series, calculating k_i, and finally forming and feeding back u.

Results

The linearized equations of motion given by Eq. (4) for a spinning symmetrical satellite in an elliptic orbit have been simulated on a digital computer. An example case where the orbital eccentricity $\epsilon = 0.5$, the inertia ratio $K = 1.0$, and the spin parameter $\alpha = -1.0$ is considered here. Using these values a Floquet analysis was done to determine the characteristic exponents and hence the stability of the uncontrolled motion. The exponents where

$$\rho_1 = 0.6718 \qquad \rho_3 = 0.0 + 0.3728i$$
$$\rho_2 = -0.6718 \qquad \rho_4 = 0.0 - 0.3728i$$

Since ρ_1 is positive the uncontrolled motion is unstable. The $F^{-1}(\tau)$ matrix was formed and $r_1(\tau)$ was determined and expressed in a Fourier series. The Fourier series for $r_1(\tau)$ has in this case a dc component given by

$$a_{0_1} = 0.6197$$

The control necessary therefore to change just ρ_1 is given by

$$u = [k_1, 0, 0, 0] F^{-1}(\tau) \bar{x} \tag{27}$$

where k_1 is given by $k_1 = (\rho_{1_{\mathrm{CL}}} - \rho_1)/a_{0_1}$. A value of $k_1 = -2.0$ was chosen and a Floquet analysis of the closed-loop system yielded

$$\rho_{1_{\mathrm{CL}}} = -0.5677 \qquad \rho_{3_{\mathrm{CL}}} = 0.0 + 0.3728i$$
$$\rho_{2_{\mathrm{CL}}} = -0.6718 \qquad \rho_{4_{\mathrm{CL}}} = 0.0 + 0.3728i$$

These results agree to four places with the results predicted by the theory. Only $\rho_{1_{\mathrm{CL}}}$ is different from the open-loop values and it is as predicted.

Conclusions

In summary, the modal control of a system of linear differential equations with periodic coefficients has been considered. General results for determining single-mode control have been derived. The general results have been demonstrated in controlling the attitude motion of a spinning symmetric satellite.

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Recursive Relationships for Body Axis Rotation Rates

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Introduction

IN many algorithms and areas of analysis in flight dynamics and flight control systems, both body axis rotation rates and Euler angle rates are measured and/or used to control the motion of the vehicle. There are a number of Euler sequences (12 possible sets for three Euler rotations) any one of which can be used to describe a problem or may be required by hardware definition. Since there are quite a number of sequences possible and each equation relating the body axis rotation rates to the Euler rates is unique for that sequence, these relationships are not written out in the literature. The following analysis provides the engineer with a method of computing all these relationships and extends the equations to n Euler rotations. These equations lend themselves immediately to computer applications and can also be used to write out a particular Euler sequence when the computing equations are needed.

The transformation equation,

$$x = a\bar{x} \tag{1}$$

transforms vectors from the moving system ($\bar{x}$) to the inertial (stationary) system (x) at a particular instant in time. The transformation matrix can be constructed from n single-axis Euler rotations,

$$a = R_1 R_2 R_3 \ldots R_n \tag{2}$$

In most applications three or less Euler rotations are used, i.e., $n \leq 3$; however, the Space Shuttle IMU system uses a four-axis gimbal, thus requiring four Euler rotations to describe the transformation matrix from the navigation mounting base on the Space Shuttle to the stable member (inertial platform). The transformation matrix is orthogonal and has the properties,

$$a^T a = a a^T = I \tag{3}$$

where I is the unity matrix.

Differentiating Eq. (3),

$$\dot{a}^T a + a^T \dot{a} = \dot{a} a^T + a \dot{a}^T = 0 \tag{4}$$

then

$$\dot{a}^T a = -a^T \dot{a} \quad \text{and} \quad \dot{a} a^T = -a \dot{a}^T \tag{5}$$

or

$$a^T \dot{a} = -(a^T \dot{a})^T \tag{6}$$

which states that $a^T \dot{a}$ is skew symmetric. From other analyses we know that

$$\dot{a} = a\bar{W} \tag{7}$$

where $\bar{W}$ is the skew symmetric matrix of body axis rotation rates ($\bar{\omega}$), namely,

$$\bar{W} = \begin{bmatrix} 0 & -\bar{\omega}_3 & \bar{\omega}_2 \\ \bar{\omega}_3 & 0 & -\bar{\omega}_1 \\ -\bar{\omega}_2 & \bar{\omega}_1 & 0 \end{bmatrix} \tag{8}$$

Further if we differentiate Eq. (2),

$$\dot{a} = \dot{R}_1 R_2 R_3 \ldots R_n + R_1 \dot{R}_2 R_3 \ldots R_n + R_1 R_2 \dot{R}_3 \ldots R_n + \ldots + R_1 R_2 R_3 \ldots \dot{R}_n \tag{9}$$

Note that, like Eq. (7),

$$\dot{R}_n = R_n W_n \tag{10}$$

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